ME 305 Fluid Mechanics I

Part 2

Fluid Statics

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Fluid Statics

- Fluids can NOT remain at rest under the presence of shear stress.
- In other words, fluids at rest can NOT support any shear.
- For static fluids we can only talk about normal stress which is equal to pressure.
- Determining the pressure distribution within a static fluid is the main task here.
- Applications include
  - Pressure distribution in still atmosphere and oceans.
  - Pressure measurement using manometers.
  - Forces acting on submerged solid bodies.
  - Bouyancy and stabilitiy of floating bodies.
- Fluids in rigid body motion are also free of shear forces and their analysis is very similar to that of static fluids. They'll be studied later in ME 305.

http://www.cromwell.org.nz/aerial_photos/pages/Clyde Dam_jpg.htm

Pressure

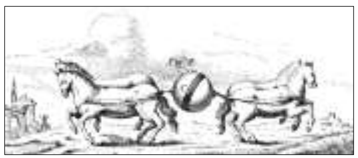
- For a fluid at rest, pressure is defined as the normal force acting per unit area exerted on a surface immersed in the fluid.
- It is due to the bombardment of the surface with the fluid molecules.

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101.325 \text{ kPa} = 1.01325 \text{ bars} = 14.7 \text{ psi}$$

- Atmospheric pressure that we feel is due to the air column sitting on top of us.
- It is quite high (~10^5 Newton per m^2 or ~10 tones per m^2).

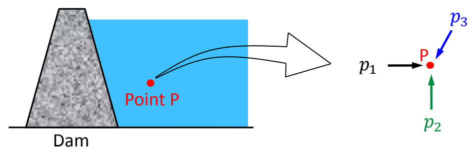


Famous Magdeburg experiment that demonstrates the power of the atmospheric pressure

https://en.wikipedia.org/wiki/Magdeburg_hemispheres

Direction Dependency of Pressure

Exercise : For a fluid at rest, pressure at a point is independent of direction, which is known as Pascal's Law. Find and study its derivation in your fluid mechanics book.

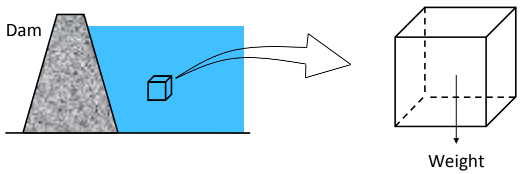


$$\boxed{p_1 = p_2 = p_3 = \cdots}$$

- In a moving fluid there will be both static and dynamic pressure definitions. Static pressure will be defined in a special way. It'll be a bit tricky.

Pressure Variation in a Static Fluid

- As we dive deep into the sea we feel more pressure in our ears.
- When we travel to high altitudes atmospheric pressure decreases.
- Following fluid element in a static fluid is not moving because no net force acts on it.



- For static fluids $\sum \text{Forces} = 0 \;\rightarrow\; \text{Weight} + ??? = 0$

Net pressure force

Pressure Variation in a Static Fluid

Exercise : In a static fluid with weight being the only body force, derive the following hydrostatic force balance.

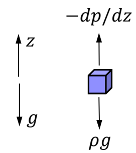
$$\underbrace{-\nabla p}_{\text{Net pressure force per unit volume}} + \underbrace{\rho \vec{g}}_{\text{Weight per unit volume}} = 0$$

- It is common to select $\vec{g}$ in the negative z direction, i.e. $\vec{g} = -g\,\vec{k}$ ($g = 9.81$)

  In this case the above equation reduces to $\boxed{\dfrac{dp}{dz} + \rho g = 0}$

  Pressure only changes in the z direction, not x or y.

- Remember that $\rho g = \gamma$ (specific weight)

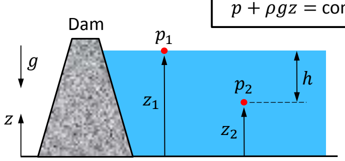


Pressure Variation in a Static Fluid (cont'd)

$\dfrac{dp}{dz} = -\rho g \;\rightarrow\; p(z) = ?$ To evaluate $p(z)$, i.e. to perform the integration, we need to know how ρ and g change with z.

- Consider g to be independent of z. At sea level it is 9.807 m/s^2 and at 14 km altitude it is 9.764 m/s^2 (less than 0.5 % change).
- Also for simplicity let's consider constant density (incompressible fluid).
- If ρg is constant $dp/dz = -\rho g$ can be integrated to give

$$\boxed{p + \rho g z = \text{constant}}$$

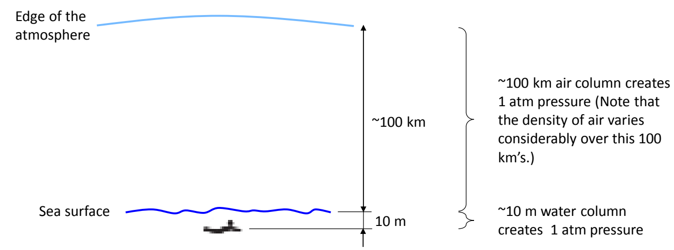


$$p_1 + \rho g z_1 = p_2 + \rho g z_2$$
$$p_2 = p_1 + \rho g (z_1 - z_2)$$
$$p_2 = p_1 + \rho g h$$

- As we go down in a constant density fluid pressure increases linearly with depth.

Pressure Variation in a Static Fluid (cont'd)

Exercise : How deep in the sea should you dive to feel twice the atmospheric pressure?



Pressure Variation with Variable Density

- If $\rho \neq$ constant, we need a $\rho(z)$ relation to integrate $\dfrac{dp}{dz} = -\rho g$

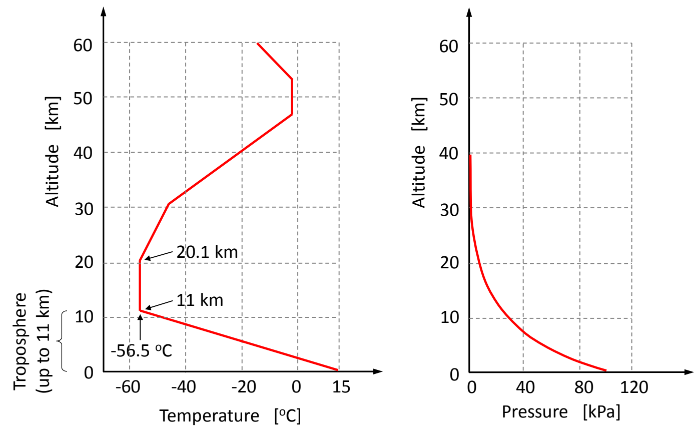
Exercise : According to U.S. Standard atmosphere model (see the next slide), temperature within the first 11 km of the atmosphere drops linearly as $T = T_0 + Bz$, where the temperature at the ground is $T_0 = 288\ \mathrm{K}$ (15 °C) and the temperature lapse rate is $B = -0.0065\ \mathrm{K/m}$.

a) Considering that the pressure at the ground level ($z = 0$) is equal to $101325\ \mathrm{Pa}$ and treating air as an ideal gas, obtain the pressure variation as a function of z within the first 11 km of the atmosphere.

b) Use the equation derived in part (a) to calculate the atmospheric pressure at the peak of Mount Everest ($z = 8848\ \mathrm{m}$). Does your result match with the second figure of the next slide?

c) How much percent error would there be in the previous calculation if we assume atmospheric air to be isothermal at i) ground temperature of 15 °C, ii) an average temperature of -14 °C ?

U.S. Standard Atmosphere Model



Pressure Variation with Variable Density (cont'd)

Exercise : In 1960 Trieste sea vessel carried two oceanographers to the deepest point in Earth's oceans, Challenger Deep in the Mariana Trench (10,916 m). Designers of Trieste needed to know the pressure at this depth. They performed two calculations. First they considered the seawater to be incompressible with a density equal to the value at the ocean surface, which is 1020 kg/m^3. Then they considered the compressibility of seawater using a modulus of elasticity of 2.07x10^9 Pa. Taking the atmospheric pressure at the ocean surface to be 101.3 kPa, calculate the percent error they made in the calculation of pressure at h = 10,916 m when they considered seawater to be incompressible.

http://en.wikipedia.org/wiki/Bathyscaphe_Trieste

Also read about James Cameron's 2012 dive at the Challenger Deep.

http://www.deepseachallenge.com

Absolute vs Gage Pressure

- Absolute pressure is measured with respect to complete vacuum.
- Certain pressure measuring devices measure pressure with respect to the ambient pressure, which is usually the atmospheric pressure. This is called gage pressure.
- Gage pressure is commonly used when we want to get rid of the atmospheric pressure effect.
- When your car's manual says that you need to inflate the tires to 30 psi, it is actually trying to say 30 psi gage (30 psi g). If the local atmospheric pressure is 95 kPa, absolute pressure of air inside the tires would be

$$\text{Absolute pressure in the tire} = 30\ \text{psi}\left(\frac{101.3\ \text{kPa}}{14.7\ \text{psi}}\right) + 95\ \text{kPa} = 301\ \text{kPa}$$

Tire pressure is 301 kPa absolute

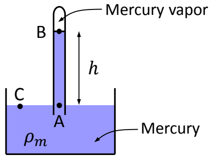
p_{atm} = 95 kPa

Pressure gage reads 30 psi g = 206 kPa g

Pressure Measuring Devices - Mercury Barometer

- In 1643 Toricelli demonstrated that atmospheric pressure can be measured using a mercury barometer. Greek word "baros" means weight.

$p_{atm} = ?$



$$p_B = p_{\text{mercury vapor}} \approx 0$$
$$p_A = p_C = p_{atm}$$
$$p_A = p_B + \rho_m g h$$
$$p_{atm} = \rho_m g h$$

- For $p_{atm} = 101{,}325$ Pa and $\rho_m = 13{,}595$ kg/m³ mercury rise will be $h = 0.76$ m.
- mmHg is another unit used for pressure. It gives the pressure difference across a 1 mm mercury column.

$$1\ \text{mmHg} = \overbrace{\left(13595\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(10^{-3}\ \text{m}\right)}^{\rho_m g h} = 133.4\ \text{Pa}$$

1 atm = 101,325 Pa = 760 mmHg

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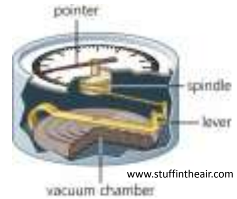
Pressure Measuring Devices - Aneroid Barometer

- Aneroid means "without fluid".
- Aneroid barometer measures absolute pressure.
- It has a vacuumed chamber with an elastic surface.
- When pressure is imposed on this surface, it deflects inward.
- Due to this deflection the needle will rotate and show the pressure.
- After proper calibration, a barometer can also be used as an altimeter, to measure altitude. Below a certain altitude, atmospheric pressure decreases 1 millibar for each 8 m of ascent.
- To read more about the aneroid barometer

  http://www.bom.gov.au/info/aneroid/aneroid.shtml



www.stuffintheair.com


www.free-online-private-pilot-ground-school.com

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Pressure Measuring Devices - Bourdon Gage

- Measures the gage pressure. Patented at 1849.
- A bent elliptical tube is open and fixed at one end, and closed but free to move at the other end.
- When pressure is applied to this tube it deflects and the pointer connected to its free end shows the gage pressure (pressure with respect to the atmospheric pressure outside of the tube).
- When the tube is disconnected the pointer shows zero.
- It can be used for the measurement of liquid and gas pressures upto 100s of MPa.


www.discoverarmfield.co.uk

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Pressure Measuring Devices - Pressure Transducer

- Pressure transducers generate an electrical signal as a function of the pressure they are exposed to.
- They work on many different technologies, such as
  - Piezoresistive
  - Piezoelectric
  - Capacitive
  - Electromagnetic
  - Optical
  - Thermal
  - etc.


http://www.lefoo.com

- They can be used to measure rapid pressure fluctuations in time.
- Differential types can directly measure pressure differences.

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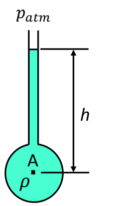
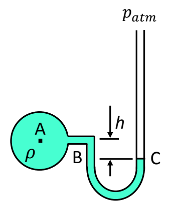
Manometers

- Manometers are used to measure pressure differences using liquid columns in tubes.
- Working principles are
  - any two points at the same elevation in a continuous liquid have the same pressure.
  - pressure increases as $\rho g h$ as one goes down in a liquid column.

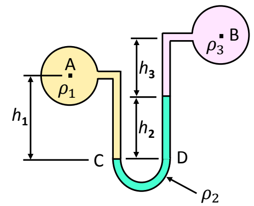


$p_A = p_{atm} + \rho g h$

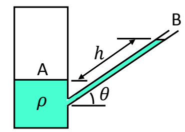


$p_A + \rho g h = p_B$
$= p_C = p_{atm}$

Manometers (cont'd)



$p_A + \rho_1 g h_1 = p_C = p_D$
$= p_B + \rho_3 g h_3 + \rho_2 g h_2$



$p_B + \rho g h \sin(\theta) = p_A$

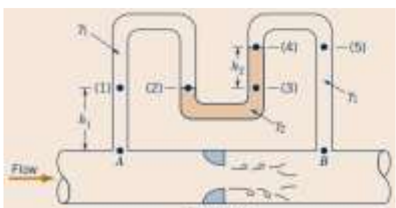
Exercise : What's the advantage of using an inclined manometer instead of a vertical one?

Manometers (cont'd)

Exercise : The volume rate of flow, Q, through a pipe can be determined by means of a flow nozzle located in the pipe as illustrated below. The nozzle creates a pressure drop, $p_A - p_B$, along the pipe that is related to the flow through the equation $Q = K\sqrt{p_A - p_B}$, where K is a constant depending on the pipe and nozzle size. The pressure drop is frequently measured with a differential U-tube manometer.

a) Determine an equation for $p_A - p_B$ in terms of the specific weight of the flowing fluid, the specific weight of the gage fluid, and the various heights indicated.

b) For $\gamma_1 = 9.8\ \mathrm{kN/m^3}$, $\gamma_2 = 15.6\ \mathrm{kN/m^3}$, $h_1 = 1$ m and $h_2 = 0.5$ m, calculate $p_A - p_B$.

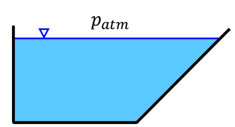


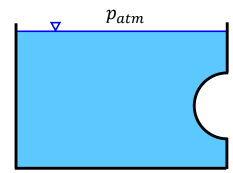
Munson's book

Hydrostatic Forces Acting on Submerged Surfaces

- Pressure force always acts perpendicular to a surface in a compressive manner.

Exercise : Show the variation of pressure force acting on the walls of the following containers. Pay attention to both magnitude and direction.

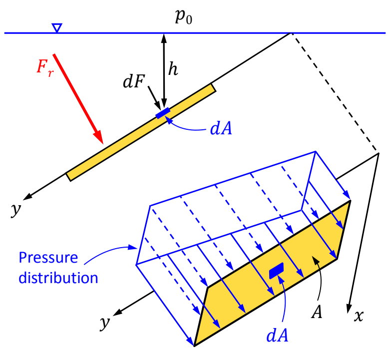




- The task is to find the resultant pressure force acting on a submerged surface and point of application of the resultant pressure force.
- Different techniques can be used such as :
  1. Direct Integration Method
  2. Pressure Prism Method
  3. Force Component Method

Direct Integration Method

- This general technique can be used to calculate the resultant pressure force on planar or curved surfaces.
- Integrate the pressure variation on a surface to get the resultant pressure force F_r.



- Planar plate is on the xy plane.
- We're interested in the pressure force acting on its top surface.
- Differential force dF acts on the differential area dA.

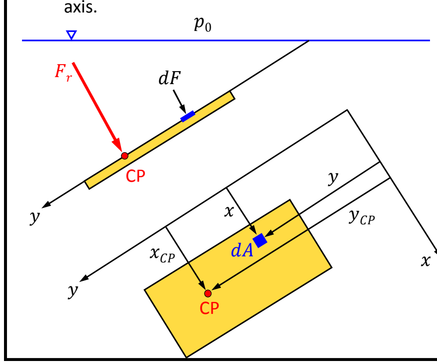
$$dF = p\,dA = (p_0 + \rho g h)\,dA$$

- Integrate dF over the plate area to get the resultant force

$$F_r = \int_A p\,dA$$

Direct Integration Method (cont'd)

- F_r acts through a point called center of pressure (CP).
- Coordinates of CP are calculated by equating the moment created by the distributed pressure force along an axis (x or y) to the moment created by F_r along the same axis.



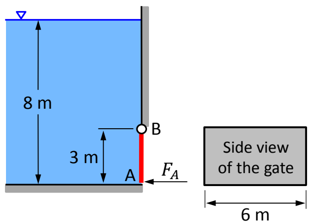
$$F_r = \int_A p\,dA$$

$$x_{CP}\,F_r = \int_A x\,p\,dA$$

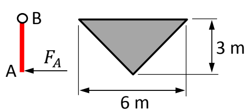
$$\underbrace{y_{CP}\,F_r}_{\text{Moments created by the resultant pressure force}} = \underbrace{\int_A y\,p\,dA}_{\text{Moments created by the distributed pressure force}}$$

Exercises for Direct Integration Method

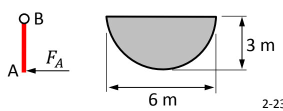
Exercise : Rectangular gate of size 6 m × 3 m is hinged along B and held by the horizontal force F_A applied at A. Calculate the force F_A required to keep the gate closed.



Exercise : Solve the previous problem by considering a triangular gate as shown below.

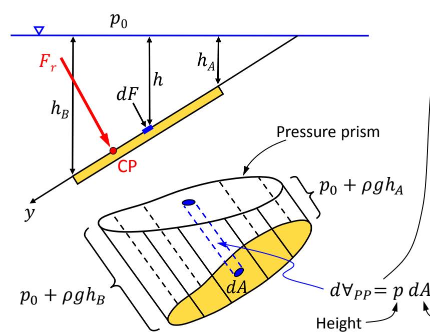


Exercise : Solve the previous problem by considering a semi-circular gate as shown below.



Pressure Prism (PP) Method

- This is an alternative (and sometimes easier) technique to calculate hydrostatic forces acting on submerged planar surfaces (not used for curved surfaces).
- Consider an imaginary prism with the planar surface of interest being its base and the amount of pressure acting on the surface being its height.



$$dF = p dA = d\forall_{PP}$$

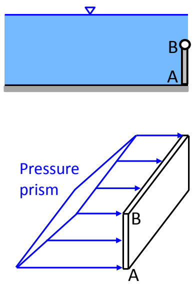
$$F_r = \int_A dF = \int_A d\forall_{PP}$$

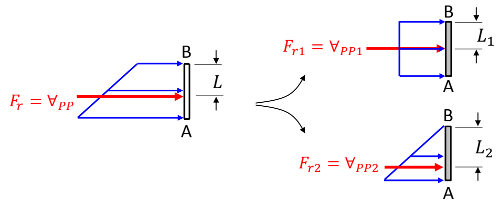
$F_r = \forall_{PP} =$ volume of the pressure prism

F_r passes through the centroid of the pressure prism.

Pressure Prism (PP) Method (cont'd)

- PP method is easy to use if $\forall_{PP}$ is easy to calculate.
- If the surface shape is complicated such that evaluation of $\forall_{PP}$ and/or its centroid requires integration, then the PP method has no advantage over the direct integration method.
- You can divide the pressure prism into sub volumes for ease of calculation.



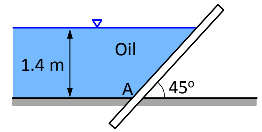


$F_r = F_{r1} + F_{r2}$, $F_r\, L = L_1\, F_{r1} + L_2\, F_{r2}$

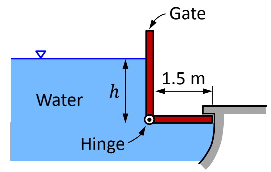
Exercises for PP Method

Exercise : Solve the problems of slide 2-22 using the PP method.

Exercise : The wall shown has a width of 4 m. Determine the total force on the wall due to oil pressure. Also determine the location of the center of pressure from point A along the wall. Density of oil is 860 kg/m³.



Exercise : (Fox's book) As water rises on the left side of the L-shaped gate, it will open automatically. Neglecting the weight of the gate, at what height h above the hinge will this occur? How will the result change (increase, decrease or no change) if the mass of the gate is considered?

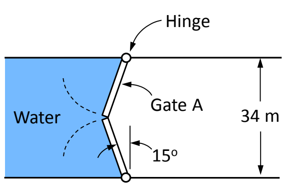


Exercises for PP Method (cont'd)

Exercise : (Fox's book) Gates in the Poe Lock at Michigan, USA close a channel, which is 34 m wide and 10 m deep. The geometry of one pair of gates is shown below. Each gate is hinged at the channel wall. When closed, edges of the gates are forced together by the water pressure. Evaluate

a) the force exerted by water on gate A.

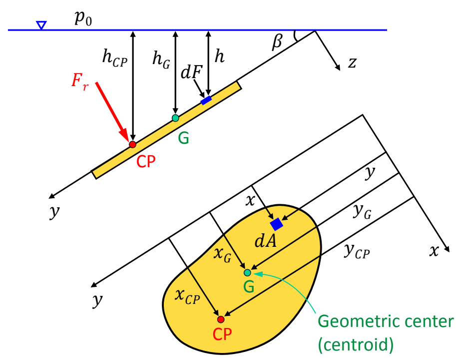
b) the reaction forces on the hinges.



Top View

Exercises (cont'd)

Exercise : Show that for a submerged planar surface resultant pressure force is equal to the pressure at the geometric center of the surface multiplied by the surface area.



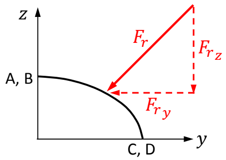
$F_r = p_G A$

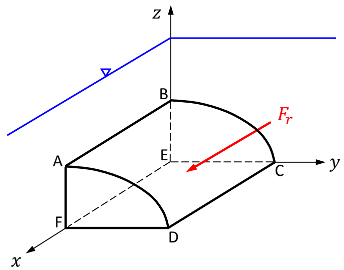
In general, points G and CP are not the same.

Exercise : Think about a case for which points G and CP are the same.

Forces Acting on Submerged Curved Surfaces
Force Component (FC) Method

- Consider the curved surface ABCD shown below.
- For simplicity it is aligned such that the net pressure force acting on it only has y and z components.
- Direct integration method can be used to calculate F_r and its point of application.
- Or the FC method can be used to calculate F_{ry} and F_{rz} separately.





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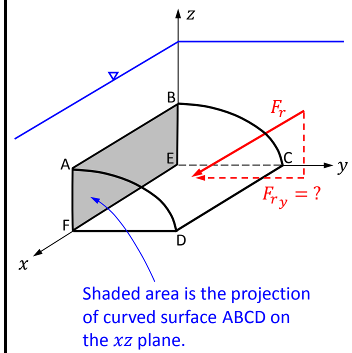
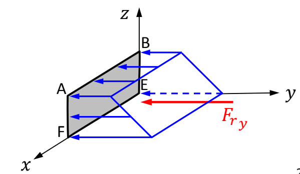
Force Component Method (cont'd)



- Horizontal component F_{ry} is equal to the force acting on the projected planar surface ABEF.

Exercise: Prove the above.

- Because ABEF is a planar surface, PP method can be used to calculate F_{ry} and its point of action.



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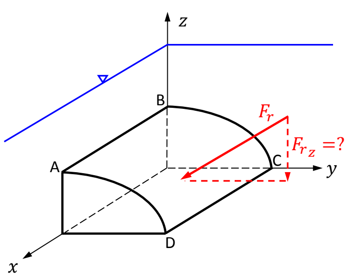
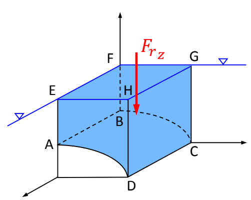
Force Component Method (cont'd)



- F_{rz} is the weight of the liquid that will fill the volume between the curved surface and the free surface, i.e. volume ABCDEFGH shown below.

Exercise: Prove the above.

- F_{rz} acts through the centroid of this volume.



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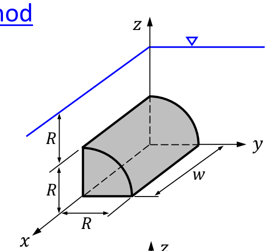
Exercises for FC Method

Exercise : Calculate the pressure force due to liquid acting on the curved surface of the shown quarter cylinder. Also determine the center of pressure.

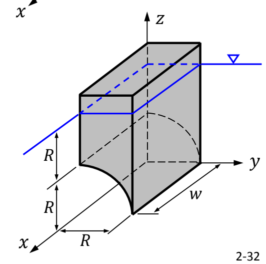
a) Use direct integration method.

b) Use force component method.



Exercise : One puzzling detail about the force component method arises when the volume between the curved surface and the free surface is NOT completely filled with liquid.

Do you think the result of this problem is any different than the previous one?



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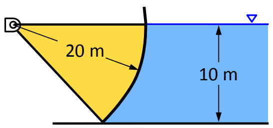
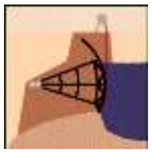
Exercises for FC Method (cont'd)

Exercise : The following Tainter gate is used to control water flow from a dam. The gate width is 35 m. Determine the magnitude and line of action of the force acting on the gate by the water. What's the advantage of using such a circular gate profile?



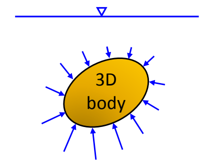


http://www.discover-net.net/~dchs/history/gate_ani.gif

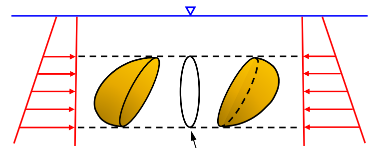
http://www.ciltug.com

Buoyancy Force

- Consider a body that is fully submerged (could be floating on the surface too) in a static fluid.
- A distributed pressure force acts all around the body.



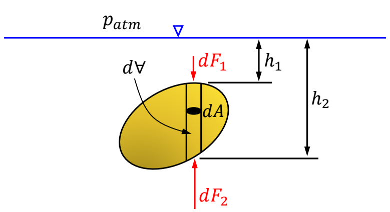
- Using the force component method we can show that the net horizontal pressure force acting on the body is zero.



Left and right parts have the same vertical projection. So the horizontal forces acting on them cancel out.

Buoyancy Force (cont'd)

- The net vertical pressure force is NOT zero. It is called the buoyancy force.



Net vertical force on dA is

$$
\begin{aligned}
dF &= dF_2 - dF_1 \\
&= (p_2 - p_1)dA \\
&= (\rho g h_2 - \rho g h_1)dA \\
&= \rho g (h_2 - h_1)dA \\
&= \rho g d\forall
\end{aligned}
$$

- Overall vertical force is obtained by integrating the above expression

$$F_{\text{vertical}} = F_{\text{bouyancy}} = \int_{\forall_{\text{body}}} \rho g d\forall = \rho g \forall_{\text{body}}$$

- Buoyancy force acting on the body is equal to the weight of the fluid displaced by submerging the body into the fluid. This is known as the Archimedes principle.

Buoyancy Force Exercises

Exercise: Use the force component method to get the following result for an arbitrarily shaped body.

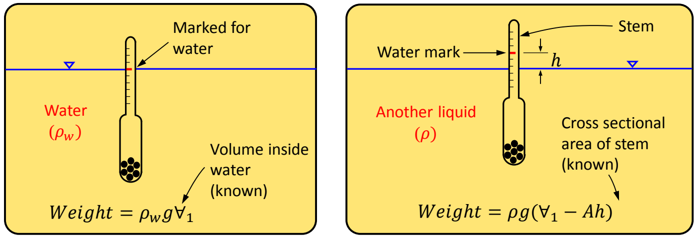
$$F_{\text{bouyancy}} = \rho g \forall_{\text{body}}$$

Exercise: The weight of a body is usually measured by disregarding buoyancy force applied by the air. Consider a 20 cm diameter spherical body of density 7800 kg/m³. What is the percentage error in calculating its weight if we neglect buoyancy?

Exercise: A 170 kg granite rock (ρ = 2700 kg/m³) is dropped into a lake. A man dives in and tries to lift the rock. Determine how much force he needs to apply to lift it from the bottom of the lake. Do you think he can do it?

Hydrometer

- Hydrometer uses the principle of buoyancy to measure the density of a liquid.
- First it is calibrated by dipping it into a liquid of known density, such as water.



$$\frac{\rho}{\rho_w} = \frac{\forall_1}{\forall_1 - Ah}$$

Movie : Hydrometer

- We read h and then calculate the unknown ρ.
- Stem may be marked so that we can directly read ρ.

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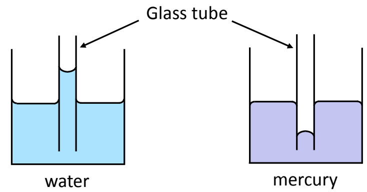
Capillarity

- When a glass tube is immersed into a liquid, which wets the surface, such as water, adhesive forces between the glass and water exceed cohesive forces in water, and water rises (capillary rise) in the glass tube.
- This vertical rise continues until the surface tension forces are balanced with the weight of the water column in the tube.
- For a non-wetting fluid, such as mercury, the force balance results in a different configuration known as capillary drop.



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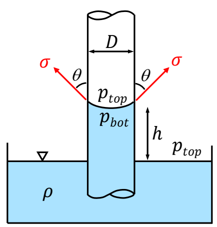
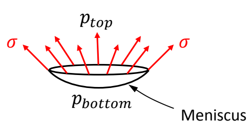
Capillarity (cont'd)

- Due to surface tension, the meniscus (free surface inside the tube) will be curved and there will be a pressure difference between the two sides of it.

$$p_{top} > p_{bottom}$$

- This pressure difference is balanced by the surface tension force.





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Capillarity (cont'd)

- Writing a vertical force balance for the meniscus

$$\sigma\, \pi D \cos(\theta) = (p_{top} - p_{bottom})\left(\frac{\pi D^2}{4}\right)$$

Force per length (σ), Length (πD)

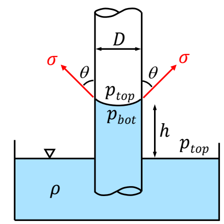
- Writing the manometer equation for the liquid between the meniscus and the free surface

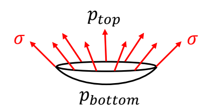
$$p_{top} = p_{bottom} + \rho g h$$

(Note that the pressure change of ambient air over the distance h is negligibly small).

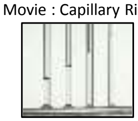
- Combining these two equations, capillary rise is given by

$$h = \frac{4\,\sigma \cos(\theta)}{\rho g D}$$





Movie : Capillary Rise



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